# THE LIFE AND TIMES OF EXTREMAL BLACK HOLES

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# THE LIFE AND TIMES OF EXTREMAL BLACK HOLES

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## Abstract

*Charged extremal black holes cannot fully evaporate through the Hawking effect and are thus long lived. Over their lifetimes, these black holes take part in a variety of astrophysical processes, including many that lead to their eventual destruction. This paper explores the various events that shape the life of extremal black holes and calculates the corresponding time scales.*

*Keywords:* Black holes, Hawking radiation, astrophysical processes

Extremal black holes contain enough charge so that their electrostatic energy compensates for their self-gravity. Because they cannot emit Hawking radiation [1] and do not evaporate, these exotic objects are often considered to live forever. Extremal black holes do not live in complete isolation, but rather inhabit a universe destined for eternal expansion. Because eternity is such a long time, we explore a collection of astrophysical processes that affect the evolution of extremal black holes and enforce their ultimate demise. Many of these processes take longer than the current age of the universe to operate and won't become urgent for quite some time.

Stellar and supermassive black holes display substantial astrophysical evidence for their existence [2]. They are thought to form through stellar death by supernova or through galaxy formation, respectively. Although we have no direct evidence for the existence of microscopic extremal black holes, it nonetheless remains possible for such objects to be forged in the early universe. Their formation time is expected to be comparable to the Planck time $\sim 10^{-43}$ sec [3]. These black holes can have either a magnetic charge or an electric charge, although we consider only the latter. (Microscopic black holes without charge would evaporate long before the present epoch.) Extremal black holes also provide an important theoretical laboratory for the study of quantum gravity (e.g., the entropy of a class of extremal black holes has been calculated from string theory [4]). Here, we consider possible evolutionary scenarios for extremal black holes and especially their ultimate fate.

For microscopic extremal black holes, the charge $Q$ required to make the horizon imaginary is $Q = M/M_{\rm pl}$, where $M$ is the mass. For simplicity, we consider the charge $Q$ to be an integer multiple of the unit electron charge $e$, so that $Q = Ze$. The black hole charge $Q$ and hence the integer $Z$ can be either positive or negative. The masses under consideration are thus of order the Planck mass $M_{\rm pl}$.

The most important processes bearing upon the evolution of extremal black holes are those that lower their charge through interactions with particles carrying charges of the opposite sign. If the black holes achieve charge neutrality, they rapidly evaporate through the Hawking effect over a Planck time.

In the early universe, extremal black holes must directly accrete particles to alter their charge. With an effective cross section comparable in size to the event horizon, $\sigma \sim {M_{\rm pl}}^{-2}$, most interactions occur at the earliest cosmological epochs when

the densities are greatest. Once extremal black holes survive the high energy environment required for their formation, direct accretion (and subsequent evaporation) is unlikely. These exotic objects are thus likely to survive until the present day.

When the cosmos is $\sim 10$ sec old, at the epoch of $e^{\pm}$ annihilation, extremal black holes drop out of kinetic equilibrium and their internal velocity dispersion falls to $\sim 1$ cm/s. Some time later at $t \approx 10^4$ yr, astrophysical structures start to form. The universe is thought to contain a substantial admixture of cold dark matter, weakly interacting particles with a mass density contribution $\Omega_{CDM} \approx 0.3$. Both dark matter and extremal black holes decouple from the background radiation field much earlier than baryons and begin to collapse before recombination (when baryonic matter collapses). The dark matter collects into self-gravitating structures that eventually become galactic halos and galaxy clusters. Extremal black holes fall into the deep gravitational potential wells carved out by the dark matter. When incorporated into galactic halos, extremal black holes exhibit dynamical behavior similar to that of the dark matter and acquire typical velocities $v/c \sim 10^{-3}$.

Once gravitationally confined to a galactic halo, extremal black holes orbit many times before suffering further interactions. Two important processes affect their long term fate: [A] Black holes with positive charge capture electrons and form bound atomic structures; similarly, black holes with negative charge interact with protons. [B] Extremal black holes pass through stars and stellar remnants, where they are captured and eventually destroyed.

The galactic disk contains an ample supply of interstellar gas that can be captured by extremal black holes. As a reference point, the recombination cross section for hydrogen is $\sigma \sim 10^{-20} - 10^{-21}$ cm$^2$ under interstellar conditions [5]. With this

cross section and typical number density $n_H \sim 1$ cm$^{-3}$, the interaction time scale $\tau = 1/n_H \sigma v \sim 10^6$ yr. Extremal black holes thus have a reasonably good chance of capturing charged particles on their passage through the galactic disk. The limiting factor is the time they spend in the inner portion of the galaxy (where the gas resides) as opposed to the far reaches of the galactic halo. Because the gas supply of the galactic disk is expected to last for $10^{13} - 10^{14}$ yr [6], extremal black holes continue to make atomic structures over this span of time.

For extremal black holes ($Z = +1$) that successfully capture electrons and form bound hydrogenic atoms, we can estimate their expected lifetime. For a positively charged black hole, the wavefunction of the electron is similar to that of the hydrogen atom. The ground state wavefunction is thus $\psi_{100} = (\pi a)^{-3/2} \exp[-r/a]$, where $a$ is the Bohr radius $a = \hbar^2/m_e e^2$. For the ground state, the probability $\mathcal{P}$ that the electron lies within the event horizon of the black hole is given by

$$\mathcal{P} = 4\pi \int_0^{R_{\rm bh}} |\psi_{100}|^2 \, r^2 dr \, \approx \frac{4}{3} (\frac{R_{\rm bh}}{a})^3 \, \approx \frac{4}{3} (\frac{2\alpha m_e}{M_{\rm pl}})^3 \sim 3 \times 10^{-73} \, .$$

Folding in the natural oscillation scale of the atom, $t_0 \sim 6 \times 10^{-17}$ sec, we find an atomic lifetime $\tau \sim 10^{49}$ yr. This time scale is much longer than the proton decay time for GUT processes ($10^{30} - 10^{40}$ yr [7]), somewhat longer than the proton decay time for gravitational processes ($10^{45}$ yr [8]), and much shorter than the evaporation time for larger astrophysical black holes ($t_{\rm evap} \approx 10^{65}$ yr $(M_{\rm bh}/M_\odot)^3$ [9]).

For an atomic structure containing a proton orbiting a negatively charged black hole ($Z = -1$), the Bohr radius is $m_P/m_e \sim 1800$ times smaller. The proton is an extended particle and the probability that the black hole lies within the proton is $\mathcal{P}_1 \approx (r_P/a)^3 \sim 6 \times 10^{-5}$. The black hole must accrete one of the proton's quarks in order to change its structure; this probability is $\mathcal{P}_2 \approx (r_{\rm bh}/r_P)^3 \sim 10^{-60}$. The

combined probability that the black holes lies inside the proton and accretes a quark is thus $\mathcal{P} = \mathcal{P}_1\mathcal{P}_2 \sim 6 \times 10^{-65}$. Combining this result with the oscillation time of the "atom", $t_0 \sim 3 \times 10^{-20}$ sec, we find an expected lifetime of $\tau \sim 10^{37}$ yr. On the smallest scale, the net result of this process is a reaction of the form: $\mathrm{q}^{+2/3}$ + $\mathrm{bh}^{-1}$ $\rightarrow$ $\mathrm{q}^{-1/3}$ + $\gamma$. Viewed from a larger scale, we see, e.g., p + $\mathrm{bh}^{-1}$ $\rightarrow$ p $\mathrm{e}^{-1}$ $\bar{\nu}$ $\gamma$.

Because the interaction cross section is low, any particular extremal black hole has a negligible chance of encountering a star during the current age of the universe ($\sim 10^{10}$ yr). The galaxy endures much longer, however, and extremal black holes can eventually interact. The stars will have long since burned out by the time black holes pass through them, so the stars are actually stellar remnants – primarily white dwarfs – for most of this time [6]. The rate at which a given extremal black hole passes through stellar remnants is given by $\Gamma = n_*\sigma_* v$, where $n_* \sim 1$ $\mathrm{pc}^{-3}$ is the number density of stars and $\sigma_*$ is their corresponding cross section. Including the effects of gravitational focusing, the cross section $\sigma_* \approx \pi R_*^2\,(1 + 2GM_*/R_* v^2) \approx 3 \times 10^{20}$ $\mathrm{cm}^2$ for a white dwarf. With its typical speed $v/c \sim 10^{-3}$, an extremal black hole encounters a white dwarf every $10^{20}$ years. This time scale is comparable to the expected galactic lifetime [6,10], the time required for the galaxy to dynamically relax and evaporate its stars into intergalactic space. Every extremal black hole should thus encounter a white dwarf about *once* during the lifetime of the galaxy.

When an extremal black hole enters a white dwarf, the binding efficiency depends on the rate at which it loses energy as it plunges through the star. This process is roughly similar to the more well studied problem of stars collecting large magnetic monopoles. In that case, main-sequence stars efficiently capture monopoles lighter than $\sim 10^{18}$ GeV and neutron stars efficiently capture all monopoles lighter than

$\sim 10^{20}$ GeV [11]. These results imply that the binding efficiency for charged black holes (with mass $\sim 10^{19}$ GeV) is close to unity for white dwarfs.

After an extremal black hole is confined to a white dwarf, it sinks to the center where the particle density is about $n \sim 10^{30}$ cm$^{-3}$ for typical remnants. The probability that an electron lies within the event horizon of a positively charged black hole is $\mathcal{P} \sim 4 \times 10^{-69}$. The oscillation time for degenerate electrons at this density is about $10^{-20}$ sec, so the time scale for electron accretion is $\tau \sim 10^{41}$ yr. (Negatively charged extremal black holes are captured with similar frequency and then interact with protons in analogous fashion.)

Over vastly longer time scales, any remaining extremal black holes can interact with electrons or positrons and form immensely large atomic structures. The time scale for electrons and positrons to form positronium in the far future of a flat universe is about $10^{85}$ yr [12]. The time required for extremal black holes to acquire either electrons or positrons is thus comparable. In an open or accelerating universe, the formation of such atomic structures is very highly suppressed. When such atomic structures are created, they are generally born in highly excited states with extremely large principle quantum numbers. The time required for these atoms to emit radiation and spiral down to their ground states is $\sim 10^{141}$ yr [12]. This time is so long compared to the decay time of the ground state ($\sim 10^{49}$ yr) that the subsequent annihilation is instantaneous by comparison.

The time scales for astrophysical processes that affect extremal black holes are summarized in Table 1, which also lists times for proton decay and black hole evaporation. Rather than living forever in stark isolation, extremal black holes experience a rich and engaging life. Charged black holes can be created in the very early

universe ($10^{-43}$ sec). Their interactions are largely insignificant until they are incorporated into galactic halos ($10^4 - 10^9$ yr). Once confined to a galaxy, extremal black holes capture charged particles and make atomic structures ($10^6 - 10^{14}$ yr). In time, the black holes accrete their charged partners and radiate away ($10^{49}$ yr). Extremal black holes are also captured by white dwarfs ($10^{20}$ yr), where they accrete charge and evaporate ($10^{41}$ yr). In a flat universe, extremal black holes that escape destruction by these means can forge gigantic atomic structures ($10^{85}$ yr), which spiral down to their ground states and eventually decay ($10^{141}$ yr). This timeline presents a rough picture for the life and relevant time scales of extremal black holes.

**Table 1**

**Time Scales for Extremal Black Holes**

| Event | Time Scale |
|---|---|
| formation of extremal black holes | $10^{-43}$ sec |
| end of kinetic equilibrium ($e^{\pm}$ annihilation) $v \to 1$ cm/s | 10 sec |
| collapse into galactic halos begins | $10^{4}$ yr |
| electron capture in dense gas (fastest rate) | $10^{6}$ yr |
| galactic halos established, $v/c \to 10^{-3}$ | $10^{9}$ yr |
| *current age of the universe* | $10^{10}$ yr |
| gas supply depleted, atomic formation ends | $10^{14}$ yr |
| accretion by white dwarfs | $10^{20}$ yr |
| *GUT scale proton decay processes* | $10^{30} - 10^{40}$ yr |
| ground state atomic decay ($bh^{-}$ $p^{+}$) | $10^{37}$ yr |
| destruction within a white dwarf ($bh^{+}$ $e^{-}$) | $10^{41}$ yr |
| *gravitational proton decay processes* | $10^{45}$ yr |
| ground state atomic decay ($bh^{+}$ $e^{-}$) | $10^{49}$ yr |
| *stellar* ($10M_{\odot}$) *black holes evaporate* | $10^{68}$ yr |
| *million solar mass black holes evaporate* | $10^{83}$ yr |
| diffuse atomic structures form in flat universe | $10^{85}$ yr |
| *billion solar mass black holes evaporate* | $10^{92}$ yr |
| diffuse atomic structures decay in flat universe | $10^{141}$ yr |

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